Complex Zeros; Fundamental Theorem of Algebra

Complex Zero: A complex zero is in the form $a+bi$ or $a-bi$.

Fundamental Theorem of Algebra: Every complex polynomial function $f(x)$ of degree $n \geq 1$ can be factored into n linear factors (not necessarily distinct) of the form $f(x)=a_n(x-r_1)(x-r_2)...(x-r_n)$, where $a_n, r_1, r_2, ..., r_n$ are complex numbers. That is, every complex polynomial function of degree $n \geq 1$ has exactly n complex zeros, some of which may repeat.

Conjugate Pairs Theorem: For any complex polynomial function, if $r=a+bi$ is a zero of f, the complex conjugate $\bar{r}=a-bi$ is also a zero of f. ★ Imaginary solutions always come in pairs.

Corollary: A polynomial f of odd degree with real coefficients has at least one real zero.

Example: A polynomial f of degree 5 has the zeros 2, 3*i*, and 4 + *i*. Find the other two zeros.

Example: Find a polynomial f of degree 4 that has the zeros 5, –3, and –2 + *i*.

Example: Find a polynomial f of degree 5 with the following zeros: 1, *i*, 4 – 2*i*.

Example: Find the complex zeros of $f(x) = 2x^4 + x^3 - 35x^2 - 113x + 65.$ Write f in factored form.

Example: Find the complex zeros of $f(x) = x^4 + 3x^3 - 19x^2 + 27x - 252.$ Write f in factored form.

Example: One of the zeros of $f(x) = x^4 - 2x^3 - 3x^2 + 10x - 10$ is $1 + i$. Find the remaining zeros.